How to measure the complex Poisson’s ratio based on the indirect method with axial-torsional Dynamic Mechanical Analysis

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The Poisson’s ratio, a fundamental mechanical property, plays a pivotal role in various industrial applications. This includes engineering and materials science, particularly when it comes to designing components for aerospace, automotive, construction, and consumer electronics. Poisson’s ratio will predict how materials deform under different types of stress and determines how materials and components behave in operation. Although the importance of the Poisson’s ratio is well-known, accurate data as a function of temperature is rarely available due to the very time-consuming measurement procedure. This article describes a novel method that enables a fast determination of the Poisson’s ratio using an axial-torsional Dynamic Mechanical Analyzer (DMA).

When a material is pulled or compressed in axial direction, it usually contracts or expands in the lateral direction. The material parameter describing this behaviour is the Poisson’s ratio ν, also known as the lateral contraction ratio. The Poisson’s ratio is a very common input for numerical simulations to predict material behaviour during mechanical loading in design, improvement or failure forecast issues.

The highest theoretical value of v is 0.5 and is approached for incompressible materials (e.g. rubbers or liquids). A Poisson’s ratio of zero means that as the material is pushed or pulled axially, it does not expand or contract laterally. This is why cork, with $\nu=0$ is used for wine bottle plugs: The plug can be pushed into the bottle relatively easily, as it does not expand laterally.

If two materials with different Poisson’s ratio are bonded together, their interface will experience stress during mechanical loading. This is because if both materials are pushed together, they will laterally expand to a different extent, resulting in shear stress at the interface.

Most polymeric materials below the glass transition temperature have Poisson’s ratios between 0.3 and 0.45 *(see Figure 1)*. Values below zero may occur in complex structures, but those are outside the scope of this article.

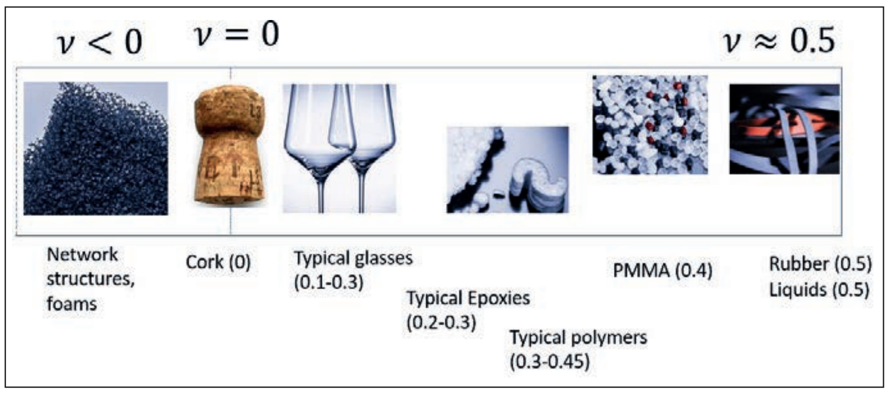

Figure 1: Poisson’s path: typical Poisson’s ratio for various materials valid under lab conditions.

Note that Poisson’s ratios mentioned in *Figure 1* are only valid under standard lab conditions, typically measured in quasi-static tests. Dependencies on temperature and time (frequency) are well-known, but not reflected in *Figure 1*.

The Poisson’s Ratio can be measured in two ways:

Direct methods

Measuring the length change in the axial and lateral direction, typically during **quasi-static testing** (Equation 1). Specimens can usually only be measured once as they are irreversibly deformed during the test.

$$\nu = \frac{d\varepsilon_{lateral}}{d\varepsilon_{axial}} \qquad \text{(Eq. 1)}$$

Indirect method

Measuring complex viscoelastic moduli E* and G* through oscillatory tests in tension and torsion. When performed within the linear viscoelastic range, several temperatures and frequencies can be measured with just one specimen. The complex Poisson’s ratio is then calculated for isotropic materials as follows:

$$\nu^*(f,T) = \frac{E^*(f,T)}{2 \cdot G^*(f,T)} - 1 \qquad \text{(Eq. 2)}$$

If different specimens were used for measurements in torsion and tension, variations and uncertainty regarding specimen dimensions may influence the accuracy of the determined complex moduli. This can, in turn, result in a higher uncertainty of the Poisson’s ratio. To overcome this, Tschoegl et al. [1] recommended to use a procedure where the same specimen can be measured in the same environment, at the same time (or frequency), and with high accuracy and precision. Until recently, indirect methods saw limited use due the lack of a device capable of recording both shear and extensional moduli [1]. In this article, we present such a device with measurement data, comparison with literature and list advantages of using the indirect approach.

Indirect measurement of Poisson’s ratio based on axial-torsional DMA

When it comes to state-of-the-art Dynamic Mechanical Analyzers that are capable to work with rotational and linear drive units, the experimental difficulties linked to the requirements of Tschoegl’s protocol [1] can be overcome easily. A specimen (rectangular or cylindrical) is clamped from the top, connected to a rotational motor, and from the bottom, connected to a linear motor. This allows measurement in torsion and tension without the need for reloading the specimen.

The MCR 702e MultiDrive with air-bearing-based rotational drive and linear drive meets the requirement to perform both measurement within a single test definition and on a single specimen. In combination with a Convection Temperature Device (CTD), a large temperature range (-150°C to 600°C) and frequency dependence of Poisson’s ratio can all be determined from loading the specimen one single time.

The most suitable measuring systems are the solid rectangular fixture (SRF) for rectangular samples or the solid circular fixture (SCF) for cylindrical samples. The preferred option are cylindrical samples measured with the SCF, as corrections to warping torsion effects are necessary for SRF [2].

Measurement of shear and extensional modulus in one test

Figure 2 shows the results obtained for a temperature ramp at different frequencies for a cylindrical polymethyl methacrylate (PMMA) sample. This reduces the operator time significantly by using the high degree of automation possible with a rheometer.

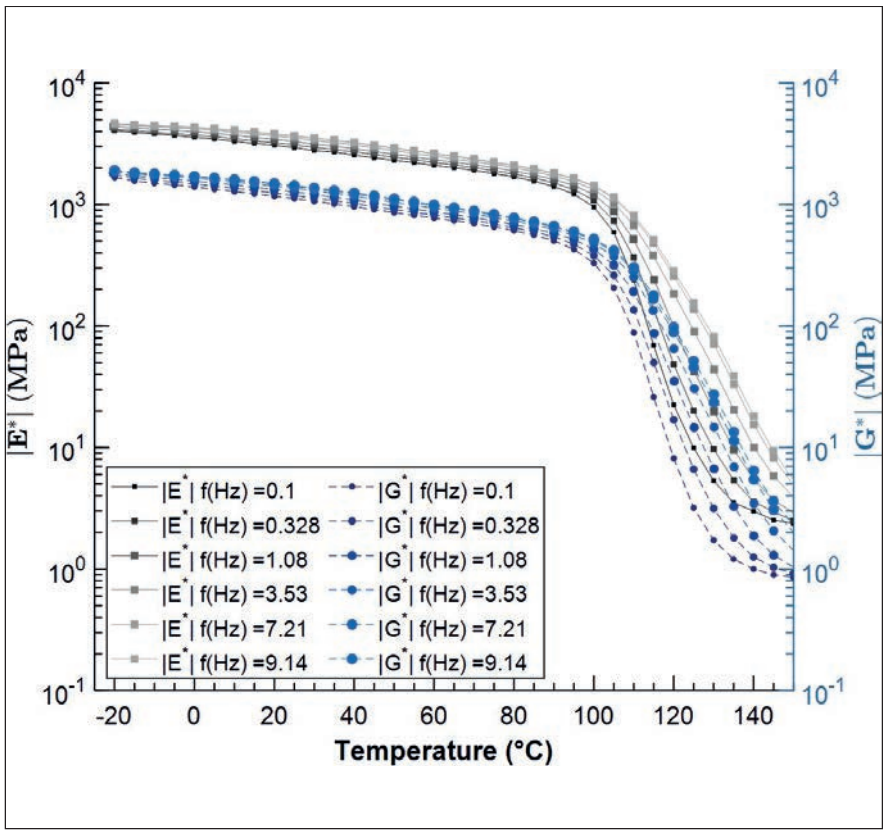

Figure 2: Complex shear modulus (blue circles) and complex extensional modulus (grey circles) data obtained on a polymethyl methacrylate (PMMA) sample at different testing frequencies (reprinted from Rodriguez Agudo et al. (2023)).

Calculation of Poisson's ratio from G* and E*

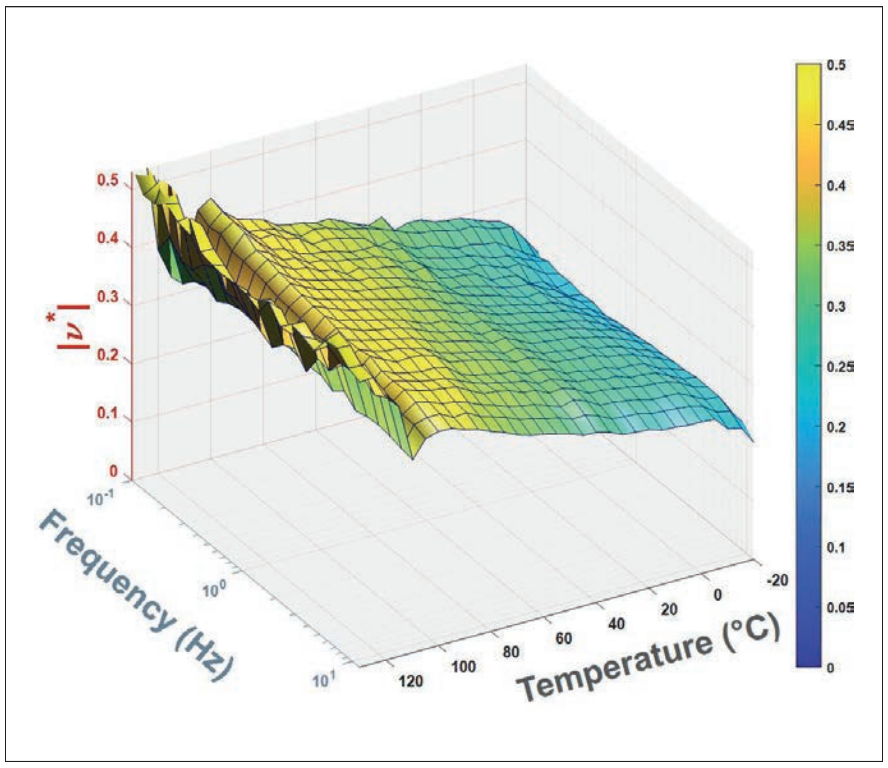

Figure 3: Poisson's ratio of PMMA calculated from modulus data in tension and torsion using Equation 2. The local minimum reflects the release of internal stresses just below the glass transition temperature through relaxation processes.

Using the results from the measurement performed in torsion and in tension, the data can be combined to calculate Poisson's ratio (*Equation 2*).

Figure 3 shows the result of the viscoelastic Poisson's ratio analysis for the modulus data presented in *Figure 2*. The results indicate that the Poisson's ratio of PMMA depends on frequency and temperature and varies between 0.2 and 0.5.

With such data, material behaviour can be predicted accounting for temperature and frequency, while single point measurements in quasi-static tests represent just one temperature. This enables a detailed, effort- and time-saving characterisation of the sample properties. A more detailed analysis of these results is provided by Rodriguez Agudo et al. [3] for standard polymers and the measurement method in general, or by Kim et al. [4] for asphalt binders.

For illustrative purposes, the complex Poisson's ratio measured at 1 Hz as a function of temperature is shown in *Figure 4* for four standard polymers: thermoplastic polyurethane (TPU), polycarbonate (PC), semi-crystalline PP, and a PC/acrylonitrile-butadiene-styrene (ABS) blend. These values were all determined by using *Equation 2* using measured values of shear and extensional modulus.

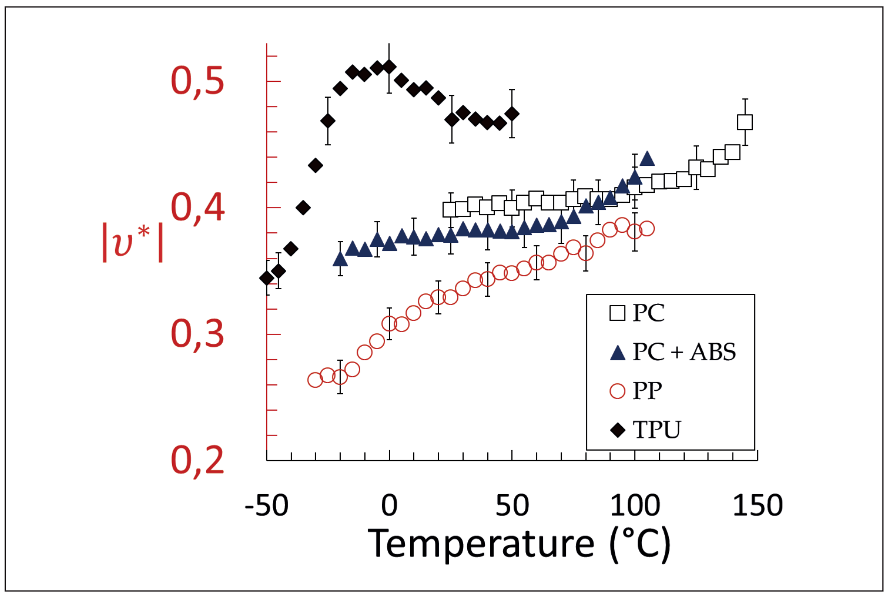

Figure 4: Poisson's ratio of multiple polymers at a testing frequency of 1 Hz. Data can be obtained reliably for a variety of polymers including blends.

At room temperature, Poisson's ratio values for PC between 0.35 and 0.42 are found in the literature for quasi-static tests [4]. Using the indirect measurement resulted in values between 0.38 to 0.41, depending on the measured frequency. The data are therefore within the expected range. For PC + ABS, values between 0.35 and 0.37 are reported [5]. In the case of the indirect method, a complex Poisson's ratio of about 0.38 has been determined, which is also within the expected values. PC and PC + ABS show very similar values for the complex Poisson's ratio up to temperatures of approximately 95°C. At this temperature, close to the main glass transition of the polymer blend, PC and ABS show significantly increasing values, diverging from PC. Differences in the influence of temperature on the Poisson's ratio can also be observed for TPU and PP.

This is evident in the broad applicability of the method and the benefit of accurately describing the Poisson's ratio specifically for each material and the dependency on temperature, in contrast to previously-used roughly determined values.

Conclusion

A state-of-the-art Dynamic Mechanical Analyzer, capable to work with rotational and linear drive units, can be used to measure the Young's modulus and the shear modulus using the same specimen. This facilitates the calculation of the viscoelastic Poisson's ratio as a function of temperature and frequency. Key advantages of this method include:

- Substantial time savings by varying environmental parameters in one experiment,
- Consistent data sets by using just one specimen for various environmental conditions
- Accurate input parameters for simulations. e.g. depending on temperature, humidity and frequency.

This new approach enables a more comprehensive characterisation of a wide variety of materials in order to optimise the simulation and development of components and assemblies. In principle, the method can also be used for the characterisation of anisotropic materials. For example, the loss factors (ratios between viscous and elastic fractions) for measurements in tension and torsion can be used to quantify the orientation-dependent differences of mechanical properties in anisotropic materials.

References

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